Mathematical Guidance for Teaching Number at Newtown School

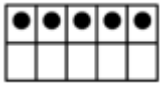
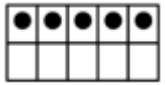
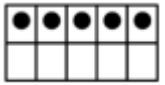
Introduction

Maths is a creative and highly interconnected discipline. It is essential to everyday life, critical to science, technology and engineering as well as being necessary for financial literacy and most forms of employment. A high-quality maths education provides the ability to reason mathematically, an appreciation of the power of maths and a sense of enjoyment and curiosity about the subject. This policy is in line with the programmes of study in the 2014 National Curriculum and the Statutory Framework for Early Years Foundation Stage 2024 and should be used to ensure consistent strategies, models and images are used across the school to support children to develop a deep understanding of number and calculation. Strategies for the four calculations are set out in the concrete, pictorial, abstract approach. This approach suggests that there are three steps necessary for children to develop understanding of a concept.

Concrete: The active stage- a child is first introduced to an idea or a skill by acting it out with real objects. This is a hands-on component and is the foundation for conceptual understanding.
Pictorial: The iconic stage- a child has sufficiently understood the hands-on experiences and can now relate them to representations such as a diagram or picture.
Abstract: The symbolic stage- a child is now capable of representing problems by using mathematical notation.
Reinforcement is achieved by going back and forth between these representations. Concrete, pictorial, abstract (CPA) concepts should not be confused as differentiation for lower, middle, higher attaining children. CPA is an approach to be used with the whole class and teachers should promote each area as equally valid. Manipulatives in particular must not be presented as a resource to support the less confident or lower attaining pupils. Children must be supported to gain a deep understanding of concepts through the CPA approach and not learn strategies as a procedure.

Mathematics Mastery

Maths teaching for mastery rejects the idea that a large proportion of people 'just can't do maths'. All children are encouraged by the belief that by working hard at maths they can succeed. Children are taught through whole-class interactive teaching, where the focus is on all children working together on the same mathematical concept. If a child fails to grasp a concept or procedure they are working on, this is identified quickly and early intervention is put in place during the lesson or before the next lesson to ensure the child is ready to move forward. Lesson design identifies the new mathematics that is to be taught: the key points, the difficult points and a carefully sequenced journey through the learning. Procedural fluency and conceptual understanding are developed in tandem because each supports the development of the other. It is recognised that practice is a vital part of learning, but the practice used is intelligent practice that both reinforces children's procedural fluency and develops their conceptual understanding. Significant time is spent developing deep knowledge of the key ideas that are needed to underpin future learning. The representations of the models and the making of connections between mathematical concepts and prior learning are emphasised, so that children develop deep learning that can be sustained. Key facts such as multiplication tables and addition facts within 10 are learnt to automaticity to avoid cognitive overload in the working memory and enable children to focus on new concepts.

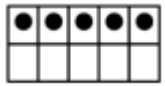
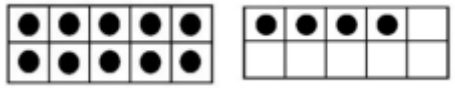
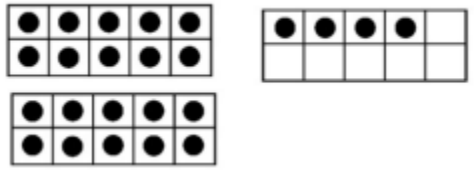
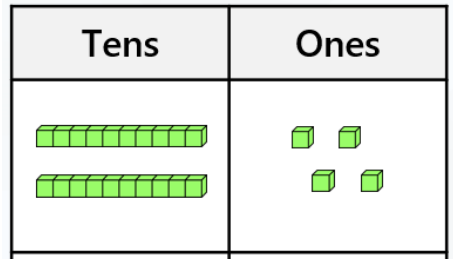
Progression in the use of manipulatives to support learning			
Pre-school	**Reception**	**Year 1**	**Year 2**
Real life objects	Real life objects	Real life objects	Real life objects
0-5 digit cards	0-9 digit cards	0-9 digit cards	0-9 digit cards
Number track to 10	Number track to 10/20		
	Number line to 10/20	Number line to 20/100	Number line to 20/100
		Hundred Square	Hundred Square
	Numbered counting stick	Counting stick	Counting stick
	Five frame Tens frame 	Tens frame 	Tens frame 
		Place value charts- 10s and ones	Place value charts
Interlocking cubes	Interlocking cubes (One colour to represent one amount)	Base 10	Base 10
	Part, part whole model	Part, part whole model	Part, part whole model
Bead strings-10	Bead strings-10,20	Bead strings-10, 20 and 30	Bead strings-10, 20, 30 and 100
Numicon shapes	Numicon shapes	Numicon shapes	Numicon shapes
Double sided counters	Double sided counters	Double sided counters	Double sided counters
Multilink	Multilink	Multilink	Multilink

Mental maths progression in line with the programmes of study in the 2014 National Curriculum and the Statutory Framework for Early Years Foundation Stage 2024:

Progression in mental maths →				
		Reception	**Year 1**	**Year 2**
Number bonds		Working with bonds for numbers up to 10.	Addition and subtraction facts within 20.	Recall and use addition and subtraction facts to 20 fluently. Derive and use related facts up to 100. E.g. 3 + 7 = 10 so know that 30 + 70 = 100
Doubling and halving			Double numbers up to 10. Halve numbers up to 20.	Double numbers up to 25. Halve numbers up to 50 (link to 2x table).
Times tables			Begin to link counting in multiples of 2, 5 and 10 to multiplication facts.	
Counting	Say names of numbers to 10. Count on and back in 1s from 0-10	Say names to 10. Count on and back in 1s from 0-20.	Count on and back in 1s from 0-100. Count in multiples of 2, 5, 10. When given a number, identify one more and one less.	Count in multiples of 2s, 3s, and 5s. Count on and back in 10s from any number.
Partitioning and place value				Recognise the place value of each digit in a two-digit number. Flexible partition 2-digit numbers in different ways e.g. 23 = 20 + 3 = 10 + 13.
Addition			Add and subtract within 20.	Add and subtract 2-digit number by one digit by counting back and counting on. Add three single digit numbers. Compensating for 9- adding 10 and taking away 1. Add near doubles.

Fundamental pre-counting and counting skills:

	Progression in teaching counting in EYFS →						
Skill	Pre-counting	Ordering	One to one correspondence	Cardinality (knowing the final number counted is the total number of objects)	Subitising (recognise small numbers without counting them)	Abstraction	Conservation of number
Description	The key focus is an understanding of the concepts **more**, **less** and **the same** and how these are related. Children at this stage develop these concepts by comparison and no counting is involved.	Count by reciting the number names backwards and forwards from any starting point.	One number word must be matched to each object. Lack of co-ordination is a source of potential error- it helps if children move the objects as they count, use large rhythmic movements or clap as they count.	Count out a number of objects from a larger collection. Know the number they stop counting at will give the total number of objects.	Children need to recognise small numbers without counting them such as spots on a dice, dots on dominoes, dots on tens frames, cards as well as small groups of randomly arranged shapes/objects.	You can count anything, visible objects, hidden objects, imaginary objects, sounds etc. Children find it harder to count things they can't move or a mix of different objects or similar objects of different sizes.	Ultimately, children need to realise that when objects are rearranged the number of them stays the same.
Ideas	Provide children with opportunities to sort groups of objects explicitly using the language of **more** and **less**. *Which group of apples has the **most**? Which group of apples have the **least**?*	Provide children with opportunities to count orally daily. Rote count so children can understand the number order and can hear the rhythm and pattern. Use a drum or clap to keep beat.	Play counting games together moving along a track, play games involving amounts such as knocking down skittles. Use traditional counting songs throughout the day.	*How many apples are there in the fruit bowl?* Allow children to physically touch the apples. Provide children with objects to point to and move as they count and say the numbers.	Provide children with opportunities to count by recognising amounts.	Provide children with a variety of physical objects, objects in pictures and sounds/actions to count.	Get a group of children to stand in different ways and ask the children how many there are repeatedly.

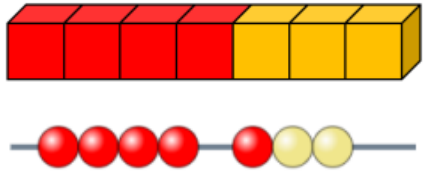
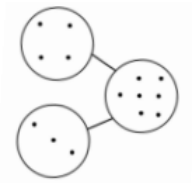
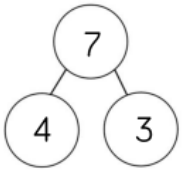
Progression in teaching Place Value			
Skill	**Understanding 10** (Pre-school and reception)	**Understanding numbers up to 20** (Year 1)	**Understanding numbers up to 100** (Year 2)
Strategies for teaching	 Tens frame is a simple maths tool which helps children • Keep track of counting • See number relationships • Learn addition to 10 • Understand place value Use the tens frame flash cards daily to ensure children recognise amounts. Use empty tens frame flash cards to fill with counters to enable children to understand number relationships. Enter the tens frame in pairs or rows. Children will clearly be able to see more than and less than 5. Setting the counters in pairs naturally allows them to see addition concepts. Other visuals to use: • Dice • Numicon • Cards • Bead strings • Cubes	Ten is the building block of our base 10 system. Children can usually read 2-digit numbers long before they understand the effect of the placement of the digit. A child may be able to read 25 and 52 and may be able to identify which is larger without understanding why the numbers are of differing values. Tens frame can provide a first step into understanding 2-digit numbers simply by the introduction of a second frame.  Other visuals to use: • Numicon • Bead strings • Place value cards • Base 10 • Money	Continue to develop place value through the use of ten frames, place value cards and base 10.   Other visuals to use: • Numicon • Bead strings • Money • Place value cards

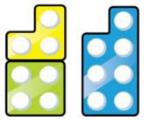
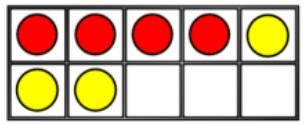
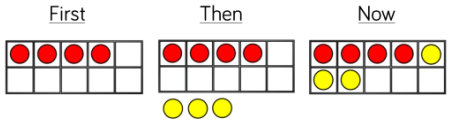
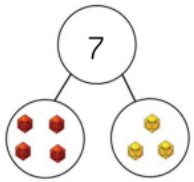
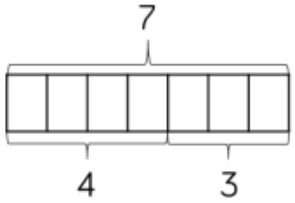
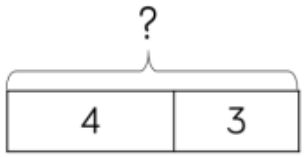
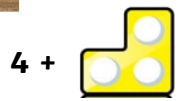
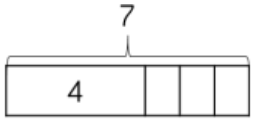
Teaching calculations progression in line with the programmes of study in the 2014 National Curriculum and the Statutory Framework for Early Years Foundation Stage 2024:

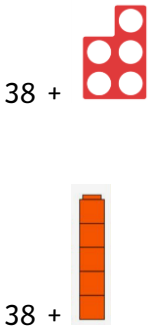
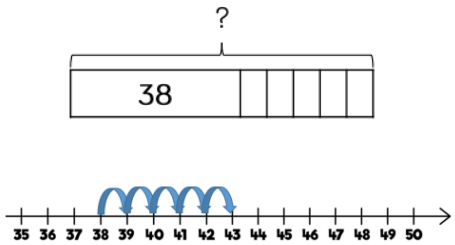
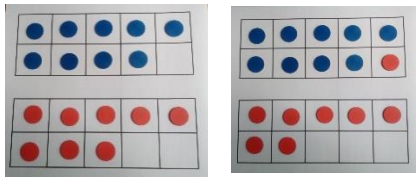
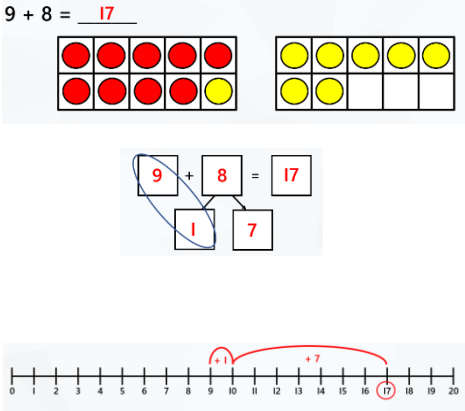
Progression in teaching calculations				
	Pre-school	**Reception**	**Year 1**	**Year 2**
Addition	In play: Combine two groups Use language more and less Solve concrete problems	Combine objects Count on using objects Use concrete and pictorial representation to record Begin to use +	Combining two parts to make a whole: part whole model Starting at the bigger number and counting on Regrouping to make 10	Adding three single digits Bridging to and through 10
Subtraction	In play: Take away objects Use language more or less Solve concrete problems	Taking away objects Counting back using objects Use concrete and pictorial representation to record Begin to use -	Taking away ones Counting back Find the difference Part whole model Make 10	Counting back Find the difference Part whole model Bridging to and through 10
Multiplication	In play: Make items fair and equal	Count equal groups of objects Count on in twos using concrete objects Doubling as repeated addition Use concrete and pictorial representation to record	Doubling Counting in multiples Arrays (with support)	Doubling Counting in multiples Arrays-showing commutative multiplication
Division		Share equally into 2 groups Understand sharing and halving as dividing by 2 Make groups of 2 Use concrete and pictorial representation to record	Sharing objects into groups Division as grouping	Division as grouping Division with arrays

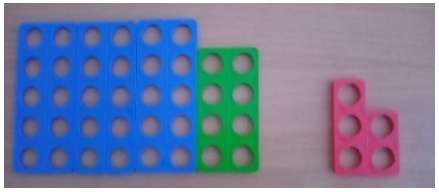
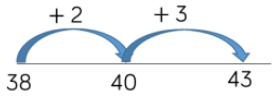
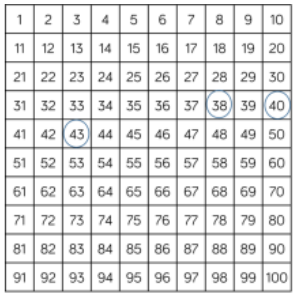
Progression in Teaching Addition and Subtraction:

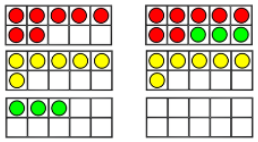
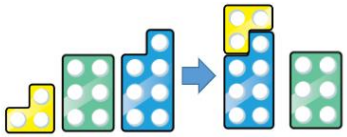
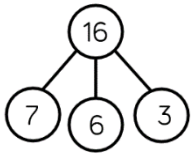
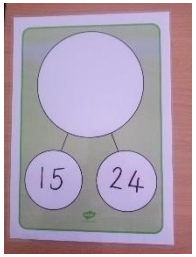
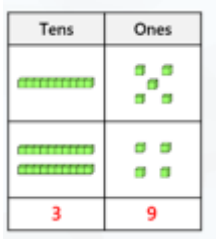
Addition

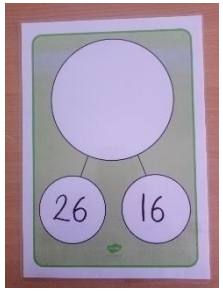
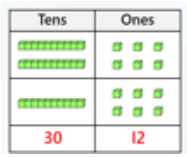
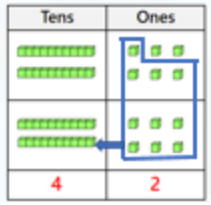
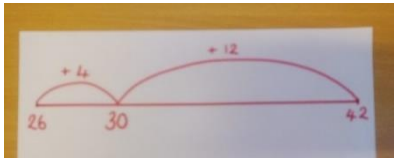
Skill	Concrete (Build it/Use it)	Pictorial (Draw it)	Abstract (Solve it) Although calculations will be shown/recorded in the concrete and pictorial stages, in the abstract stage the calculation is carried out without the use of concrete and pictorial aids.	Key vocabulary
Combining 2 groups to make a whole (aggregation) Counting sets of objects, combining them and recounting using 1:1 correspondence.	Use physical objects to add two sets of objects.  4 + 3 = 7  	Draw and use models to represent the calculation. 4 + 3 = 7    	4 + 3 = 7 10 = 6 + 4 I have 3 apples and 2 bananas. How many do I have altogether?	add, more, and, make, sum, total, altogether, one more, two more … ten more, how many more to make …? how many more is … than …?, is equal to

	   	 		
Counting on (augmentation) Where one quantity is increased by an amount. Count on from the total of the first set. Start with the largest number (commutativity).	Start at the larger number then count on. $4 + 3 = 7$   	Start at the larger number on the number line and count on in ones. $4 + 3 = 7$  	Reinforce starting with the larger number. 4 + 3 = 7 38 + 5 = 43	add, more, and, make, sum, total, altogether, plus, one more, two more … ten more, how many more to make …? how many more is … than …?, is equal to

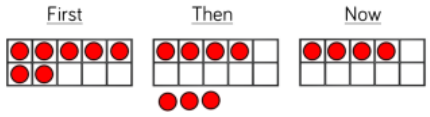
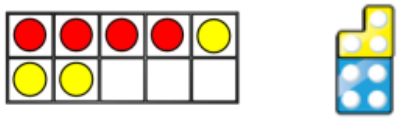
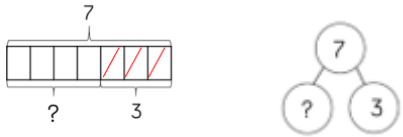
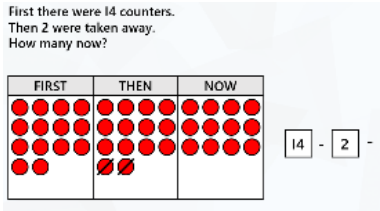
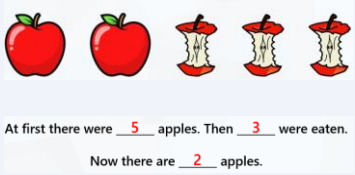
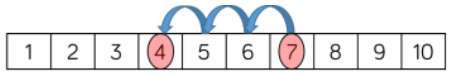
	38 + 5 = 43  	38 + 5 = 43  		
Regrouping to make 10 Rather than counting on children use their number bond knowledge to bridge 10.	Start with the larger number then use the smaller number to make 10. 9 + 8 = 17  9 + 8 = 9 + 1 = 10 10 + 7 = 17 	Draw a tens frame or number line to support making 10 before adding the rest of the number. 9 + 8 = 17  	9p + 8p = 17p *I have 9p. How much more do I need to reach 10p? How much more do I add on now?*	add, more, and, make, sum, total, altogether, plus, one more, two more … ten more, how many more to make …? how many more is … than …?, is equal to, number bonds, larger, smaller

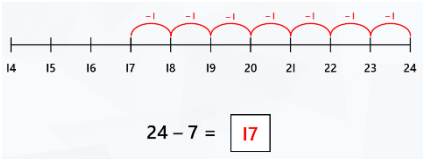
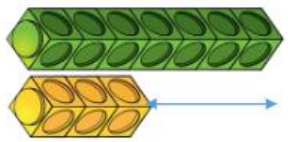
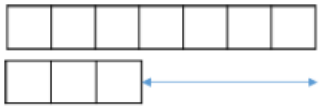
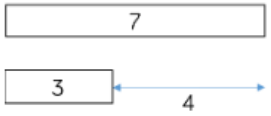
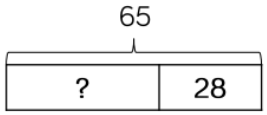
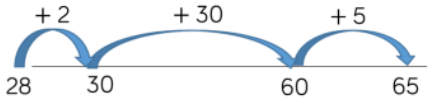
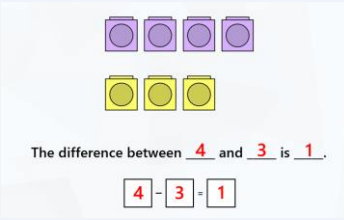
	  38 + 5 = 43  38 + 5 becomes 38 + 2 = 40 40 + 3 = 43  Is equal to 	38 + 5 = 43   		
Adding 3 single digit numbers	Manipulatives that highlight number bonds to 10 are effective when adding three 1-digit numbers.	Draw a picture of the objects or a representation of the objects.	Remind the children to look for numbers bonds to 10 or doubles to add more efficiently.	add, more, and, make, sum, total, altogether, plus, one more, two

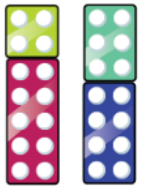
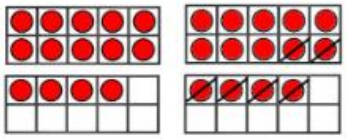
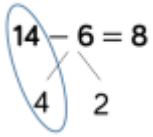
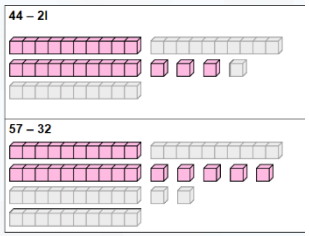
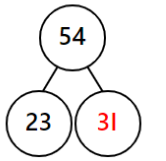
Children are encouraged to look for number bonds or doubles to add more efficiently.	$7 + 6 + 3 = 16$  	$7 + 6 + 3 = 16$ 	$7 + 6 + 3 = 16$ 10	more … ten more, how many more to make …? how many more is … than …?, is equal to, number bonds, doubles, larger, smaller
Partitioning to add Emphasis on developing deep understanding of place value and informally recording in year 2 in preparation for more formal methods in KS2.	Use base 10 and place value counters to partition the numbers. Add with no regrouping first, then regrouping No regrouping **15 + 24 = 39**  Regrouping **26 + 16 = 42**	Children can draw their own base 10 or place counters to help them solve addition calculations. No regrouping **15 + 24 = 39**  10 + 20 = 30 5 + 4 = 9 30 + 9 = 39 Regrouping **26 + 16 = 42**	Encourage children to use known facts to add *e.g. I know 1 + 2 = 3 to 10 + 20 = 30.* No regrouping **15 + 24 = 39** Regrouping using known facts **26 + 16 = 42**	add, more, and, make, sum, total, altogether, plus How many tens/ones? How many tens/ones altogether?

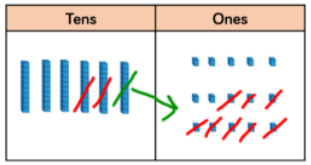
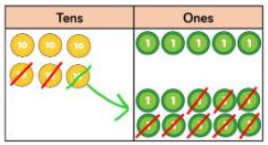
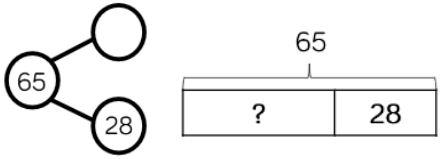
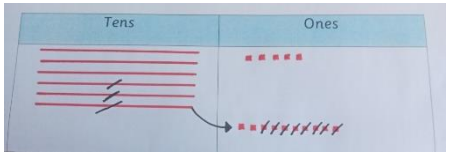
		6 + 6 = 12 20 + 10 = 30 30 + 12 = 42   	*e.g. I know that 6 + 6 = 12 so I know that my answer will have 2 ones. I will add the extra ten.*	

Subtraction

Skill	Concrete (Build it/Use it)	Pictorial (Draw it)	Abstract (Solve it)	Key vocabulary
Taking away (separation model) Where one quantity is taken away from another to calculate what is left.	Use real life objects, counters, numicon and cubes to show how objects can be taken away. 7 – 3 = 4   	Draw representations on tens frames, first, then, next boards and cross out the number to be taken away. 7 – 3 = 4  **14 – 2 = 12** 	7 – 3 = 4 First, there were 14 cakes in the shop. Then two cakes were eaten; now there are…	take away, subtract, minus, how many are left/left over? …is equal to… 
Counting back Used to subtract small numbers from larger numbers and provides a good foundation for the concept of subtraction.	Make the number on the bead string and then move the beads away while counting back. 7 – 3 = 4  Move objects whilst counting back.	Use a number line to count back. 7 – 3 = 4  **24 – 7 = 17**	Put 7 in your head, count back 4. What number? Children needs lots of practice at counting backwards. Put 24 in your head, count back 7. What number?	take away, subtract, minus, how many are left/left over? …is equal to…

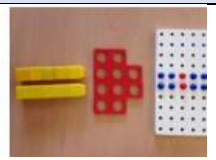
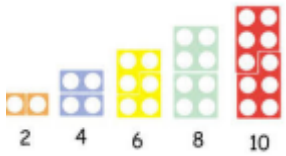
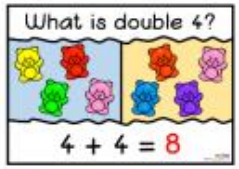
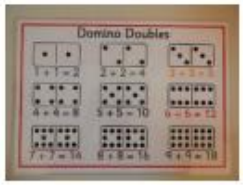
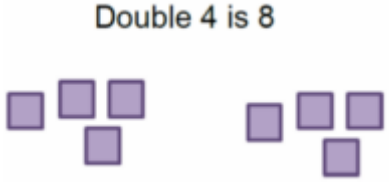
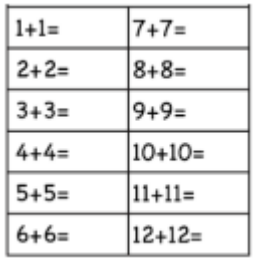
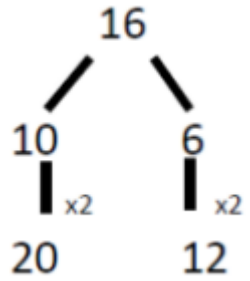
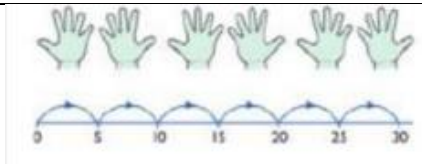
				
Finding the difference (comparison model) Two quantities are compared to find the difference.	$7 - 3 = 4$ 	$7 - 3 = 4$   $65 - 28 = 37$  	Lexie has 7 strawberries and Jake has 4. How many more does Lexie have than Jake? What is the difference between 28 and 37?	How many fewer is than…? What is the difference between? 
Regrouping to make 10 Rather than counting back	$14 - 6 = 8$	$14 - 6 = 8$	How many do we subtract to reach the next 10?	take away, subtract, minus, how many are

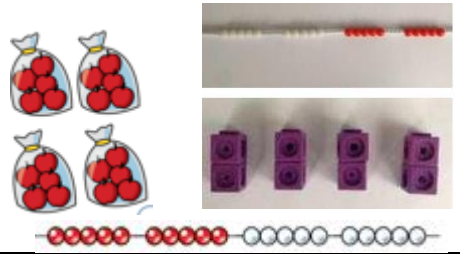
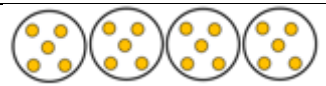
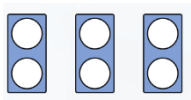
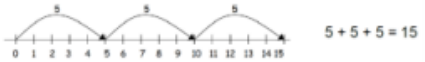
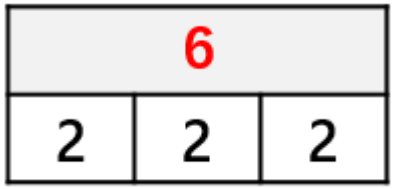
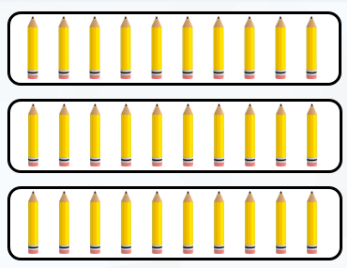
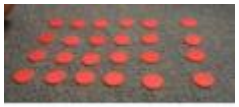
children use their number bond knowledge to bridge 10.	14 -4 = 10 10 – 2 = 8  14 -4 = 10 10 – 2 = 8 	Use a number line to take away 4 to make 10 then take away 2 to make 8 	How many do we have left to subtract? 	left/left over? …is equal to…, number bonds
Partitioning to subtract Emphasis on developing deep understanding of place value. When not regrouping, partitioning should be developed as a mental strategy rather than formal recording in columns.	Use base 10 or place value counters to physically complete the partition and subtraction. No regrouping 	No regrouping **Subtraction problem** 54 – 23 = 31 4 ones – 3 ones = 1 one. 5 tens – 2 tens = 3 tens. Part-whole model 	Encourage children to use known facts to subtract. No regrouping. e.g. I know that 5 – 2 = 3 so 50 – 20 = 30. **54 – 31 = 23**	take away, subtract, minus, how many are left/left over? …is equal to…, number bonds, partition, tens, ones

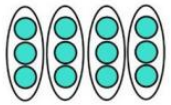
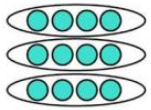
	Regrouping 65 – 28 = 37   	Regrouping 65 – 28 = 37  Draw base 10 or place value counters and cross them out.  	Regrouping Use of known facts to subtract. e.g. I know that 15 – 8 = 7 so I know my answer will have 7 ones. **65 – 28 = 37**	

Progression in Teaching Multiplication and Division:

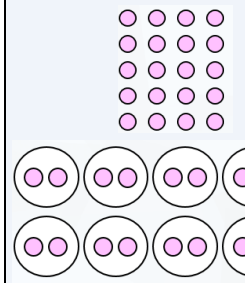
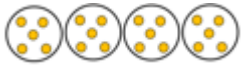
Multiplication

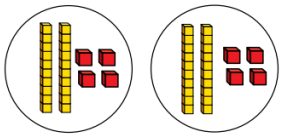
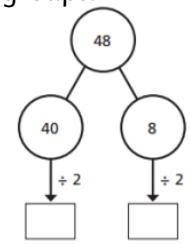
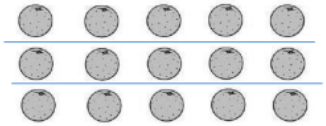
Skill	Concrete (Build it/Use it)	Pictorial (Draw it)	Abstract (Solve it)	Key vocabulary
Doubling Children should be encouraged to develop mental recall of doubles and relate to 2x table.	     Use practical activities to demonstrate doubling.	    Drawing pictures to show how to double. 	 Partition a number then double each part. 	doubling, 2x table, partition
Counting in multiples Children can use their fingers to skip count to develop an understanding of 'groups of'. Children should become	Count equal groups of objects. 	 Use number lines or pictures to support counting in multiples.	**2, 4, 6, 8, 10** **5, 10, 15, 20, 25, 30** Count in multiples aloud or write sequences of numbers.	equal groups, multiples, counting in steps, skip counting

increasingly fluent as they practise.				
Repeated addition Pupils should apply skip counting to help find the totals of repeated additions.	Use different objects to add equal objects.   2 + 2 + 2 = 6  5 + 5 + 5 = 15	Use drawings and representations to add equal groups.    	5 + 5 + 5 + 5 = 20	numeral, how many, repeated addition, equal, altogether, sum, total, sum  There are 10 in each row. There are 3 rows. 10 + 10 + 10 = 30 There are 30 altogether.
Arrays showing commutative multiplication Children should understand that the order of the multiplication	Use concrete resources to make arrays.  4 x 6 = 24 6 x 4 = 24	Draw arrays to solve multiplication calculations.  4 x 5 = 20 5 x 4 = 20	4 x 5 = 20 5 x 4 = 20 One bag holds 5 apples. How many apples do 4 bags hold?	numeral, how many, multiplication, multiply, multiplied by, multiple, groups of, lots of, times, equals, altogether, sum, total, array, row, column, number patterns,

does not affect the answer.	  3 x 5 = 15 5 x 3 = 15 4 x 6 = 24 6 x 4 = 24   4 x 3 = 12 3 x 4 = 12	 4 x 3 = 12  3 x 4 = 12		multiplication table, multiplication fact  3 x 3 = 9 3 lots of 3 = 9 3 multiplied by 3 = 9

Division

Skill	Concrete (Build it/Use it)	Pictorial (Draw it)	Abstract (Solve it)	Key vocabulary
Sharing Division is shown as sharing equally between groups.	Use a range of real life resources (including in role-play), counters, cubes and base 10 to begin to demonstrate sharing between groups. 10 shared between 2 groups  20 shared between 5 groups 	Demonstrate understanding through drawing pictures and representations. 20 shared between 5 groups  	20 ÷ 5 = 4 There are 20 apples altogether. They are shared equally between 5 bags. How many apples are in each bag?	share, between, equal groups, altogether Share 20 counters between 10 equal groups.  There are 20 altogether. There are 10 groups in total. There are 2 in each group.
Division as grouping Here division is shown as grouping. This is a good opportunity to demonstrate and reinforce the inverse relationship with multiplication.	Divide concrete resources into equal groups. 20 divided into 4 equal groups  	Drawing pictures to represent equal groups. 20 ÷ 5 = 4 	20 ÷ 5 = 4 There are 20 apples altogether. They are put in bags of 5. How many bags are there?	numeral, how many, division, divide, divided by, groups of, lots of, times, equals, altogether, sum, total, number patterns, division fact, inverse, multiplication, partition 

	48 ÷ 2 = 24 	Partition larger numbers before dividing into equal groups. 48 ÷ 2 = 24 		There are 16 cubes altogether. There are 2 in each group. There are 8 groups.
Division with arrays Use arrays of concrete manipulatives and then dots to develop more abstract concept of division.	Link multiplication and division through use of arrays.  3 x 5 = 15 5 x 3 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5	Draw arrays to solve division calculations. 4 x 5 = 20 5 x 4 = 20 20 ÷ 5 = 4 20 ÷ 4 = 5   3 x 5 = 15 5 x 3 = 15 15 ÷ 5 = 3 15 ÷ 3 = 5	Find the inverse of multiplication and division sentences by writing four linking calculations. 4 x 5 = 20 5 x 4 = 20 20 ÷ 5 = 4 20 ÷ 4 = 5	numeral, how many, division, divide, divided by, multiplication, multiply, multiplied by, multiple, groups of, lots of, times, equals, altogether, sum, total, array, row, column, number patterns, multiplication table, multiplication fact, division fact, inverse